

# Applying coalitional Game Theory by Heikki Sairanen

Consider the following politically motivated example of a game.

In the Finnish parliament members often vote according to party lines. On a most issues a simple majority is enough for a law to be passed.

After parliament election in 2007 the seats of the parliament were divided among eight parties as follows:

Party	Seats
Centre Party	51
National Coalition Party	50
Social Democratic Party of Finland	45
Left Alliance	17
Green League	15
Swedish People's Party <sup>1</sup>	10
Christian Democrats	7
True Finns	5

Source: Wikipedia

We'll assume that parliament members vote strictly together as a party. Then the vote can be seen as game with the parties as players.

We'll formulate the problem (more) generally first then apply it to the Finnish parliament.

$$N = \{1, 2, \dots, n\}$$

Voting power of each party is given as:

$$l = \{l_1, l_2, \dots, l_n\}, \text{ where } l_i \in \mathbb{N} \setminus \{0\} \forall i$$

$$v(S) = 1 \text{ if } \sum_{i \in S} l_i > C, \text{ where } C \text{ is a constant.}$$

$$v(S) = 0 \text{ if } \sum_{i \in S} l_i \leq C$$

This game is simple.

Thus we can use the following formula to calculate the Shapley value of each party.

$$\phi_i(v) = \frac{\sum_T (|T|-1)!(n-|T|)!}{n!}, \text{ where } T \text{ is a set of all winning coalitions such that } T \setminus \{i\} \text{ is a}$$

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<sup>1</sup> Includes one delegate of Bourgeois Alliance from Ahvenanmaa

losing coalition.

In our example there is no veto player thus  $Core(v) = \emptyset$ .

It is obvious that calculating all the possible coalitions by hand is too tedious. We solve this problem with a simple Matlab-script. The script can be seen in the appendix.

The results for the Finnish Parliament:

<b>Party</b>	<b>Shapley value</b>
Centre Party	0.3131
National Coalition Party	0.3036
Social Democratic Party of Finland	0.2560
Left Alliance	0.0274
Green League	0.0274
Swedish People's Party <sup>2</sup>	0.0274
Christian Democrats	0.0274
True Finns	0.0179

The results are at least interesting. It seems that almost all the small parties actually have the same miniscule voting power.

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<sup>2</sup> Includes one delegate of Bourgeois Alliance from Ahvenanmaa

## Code

*The program code for calculating the Shapley value in our example.*

```
function [shapley]=shapley(l, limit)
n = length(l);
C = generate_all_coalitions(n);

% Calculate the strengths of the coalitions
s = C*l';

shapley = zeros(n,1);
for party = 1:n
    % this is a bit inefficient
    sizeT = 0;
    for k = 1:length(s)
        row = C(k,:);
        modrow = row;
        modrow(party) = 0;
        if row(party) % party belongs to coalition
            if(row*l' > limit && modrow*l' <= limit)
                sizeT = sum(row);
                shapley(party) = shapley(party) + factorial(sizeT -
1)*factorial(n-sizeT)/factorial(n);
            end
        end
    end
end

end

end

function [c] = generate_all_coalitions(n)
c = zeros(1,n);

% there is probably a better way to do this
for k = 1:(2^n-1)
    c(k+1,:) = convert_to_binary(k,n);
end
end

function [x] = convert_to_binary(n,max)
x = zeros(1,max);
for k = max-1:-1:0
    if n - 2^k >= 0
        n = n - 2^k;
        x(k+1) = 1;
    else
        x(k+1) = 0;
    end
end
end
end
```